lar to those in which ferrites are used, with no limitations of frequencies.

For pulsed rf waves decaying gyroplasmas may be more advantageous than stationary media. With pulsed magnetic fields, isothermal gyroplasmas may be used, in particular, for rapid broad-band spectrum analysis. The limitation on the bandwidth is determined by the wave-guiding structure containing the magneto plasma. The fact that a gaseous discharge plasma can be established in any desired charge density state on microsecond or shorter time scale and be removed from that state on an equally short time scale makes the ionized gaseous medium a very flexible one whose potentialities have not been, as yet, explored to any great extent or even recognized by microwave engineers.

ACKNOWLEDGMENT

It is a pleasure to acknowledge here the contributions of M. A. Lampert, J. F. Heney, J. E. Etter, M. Gilden, L. P. McGrath, C. K. Chatterjee, and J. P. Monier who at one time or another have been associated with the author in the study of these problems.

The Three-Level Solid-State Maser*

H. E. D. SCOVIL†

Summary—**This article gives an introduction to amplification by solid-state maser techniques. Emphasis is placed on the three-level solid-state maser. The relevant physical properties of paramagnetic salts are discussed. The basis of the three-level excitation method is reviewed. Some design considerations are given. The design and performance characteristics of a particular device are mentioned.**

INTRODUCTION

MASERS (microwave amplifier by stimulated emission of radiation) offer the possibility of amplification with very low-noise figures. With suitable regeneration they may be converted into oscillators having a high degree of spectral purity.

The interaction medium consists of "uncharged" magnetic or electric dipoles. It is partially because of the lack of any charge fluctuations that these devices may exhibit low-noise characteristics. The medium is maintained in such a state that it presents negative loss or gain to incident radiation.

Beam-type masers,[1] because of their high stability, make excellent frequency standards. It is, however, just the properties that give them high stability, namely a high molecular Q and a fixed frequency, that limit their versatility as easily tunable broad-band amplifiers. In these respects solid-state masers offer advantages. The three-level maser now appears to be the most useful of the solid-state types since it amplifies in a continuous manner and its high permissible spin concentration leads to relatively large gain bandwidth products.

A review article on masers by Wittke has appeared.[2] It discusses the basic fundamentals as well as giving a brief description of each type. The purpose of the present article is to discuss in more detail the three-level solid-state maser.

The next section reviews some of the physical processes involved in the operation of the device. The following section discusses the three-level excitation method and the properties of suitable materials. Some remarks are then made about design considerations and the design and performance of a particular 6-kmc device is mentioned.

PROPERTIES AND PROCESSES OF THE MEDIUM

General Remarks

The maser medium consists of an ensemble of atomic magnetic moments or "spins" in the solid state. The individual dipoles may take up only certain discrete or "allowed" energy states as a result of interaction with crystalline electric and applied magnetic fields.

Since the medium chosen is such that the mutual interaction between dipoles is weak, the entire ensemble may be treated statistically as though all of the particles are distributed over the allowed states of an individual particle.

This system is referred to as the spin system. Interactions occur within the spin system and between the spin and the remainder of the crystal lattice as well as with a radiation field. These interactions are now discussed.

* Manuscript received by the PGMTT, September 23, 1957.
† Bell Telephone Labs. Inc., Murray Hill, N. J.
[1] J. P. Gordon, H. J. Zeiger, and C. H. Townes, "The maser—new type of microwave amplifier, frequency standard, and spectrometer," *Phys. Rev.*, vol. 99, pp. 1264–1274; August, 1955.

[2] J. P. Wittke, "Molecular amplification and generation of microwaves," PROC. IRE, vol. 45, pp. 291–316; March, 1957.

Spin-Spin Relaxation or Dipolar Interaction

Interactions within the spin system have been examined by several authors.[3,4] A heuristic treatment is given here. Consider the case of a spin surrounded by neighboring spins and subject to a magnetic field H applied in the Z direction as depicted in Fig. 1 where only one neighbor is shown. Spin (a) will be subject not only to the applied magnetic field but also to the magnetic fields of its neighbors. The field of a neighbor may be resolved into two components, one parallel to H, the other perpendicular to H.

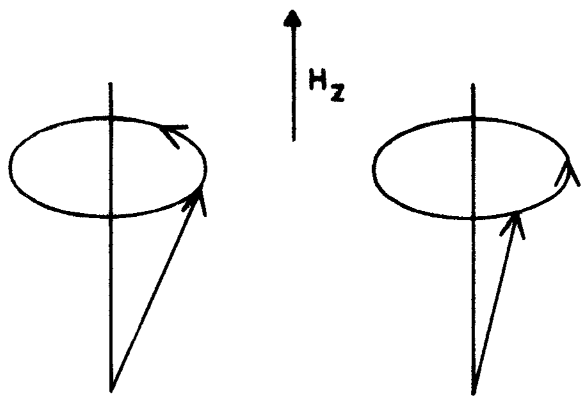

Fig. 1—Two neighboring spins (a) and (b) are depicted as precessing in an applied magnetic field. H.

The parallel component is added to H. Since the Z components of the neighbors are randomly distributed, this will cause a distribution of Larmor precession frequencies about that determined by H. This effect is known as "inhomogeneous spin-spin broadening" since its effect is closely related to the application of an inhomogeneous-magnetic field over the sample.

The transverse component of the precessing spin (b) will cause an rf magnetic field at spin (a) with a frequency equal to the Larmor frequency of (b). If spins (a) and (b) are equivalent, that is, if they have the same Larmor frequency, then this rf field is of the correct frequency to induce resonance in (a) and vice versa. Because this energy exchange reduces the lifetime of the spin states, it will broaden the resonance line. This "homogeneous spin-spin broadening" provides a mechanism whereby energy is transferred throughout the spin system. The interaction tends to keep the spin system in internal equilibrium and to destroy any coherence or phase relationships between spins in a characteristic "spin-spin relaxation time."

It is apparent that inhomogeneous broadening always occurs. Homogeneous broadening, however, is dominating only when nearby spins are equivalent. When all spins are equivalent the broadening of the resonance line by homogeneous broadening is the greater effect. The magnitude of the dipolar interaction depends upon the spin concentration. In practice it is controlled by "magnetic dilution" that is, mixed crystals are grown in which the magnetic atoms are partially replaced by isomorphous diamagnetic atoms.

Spin-Lattice Interaction

Thermal motion of the crystal lattice gives rise to time dependent crystalline electric fields. These fluctuating fields act on the orbital motion of the electron and then via the mechanism of spin-orbit coupling on the spins. This provides a means whereby energy may be transferred between the spin system and the crystal lattice which is usually treated as a heat sink at the bath temperature. This thermalizing or spin-lattice relaxation process is temperature dependent, the relaxation time increasing as the temperature is lowered.

A simple but important relationship between relaxation times may be obtained as follows. Consider a simple two-state system as depicted in Fig. 2 with $N = n_i + n_j$ particles distributed over the states and with relaxation times T_{nm} as shown. The energies of the states are E_i and E_j with $E_j > E_i$. Then

$$\frac{dn_i}{dt} = n_j/T_{ji} - n_i/T_{ij} \tag{1}$$

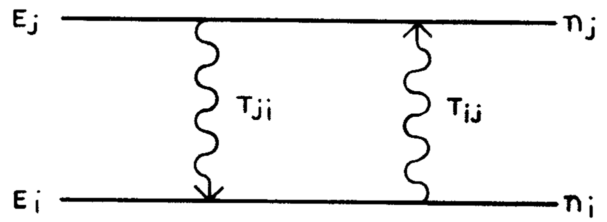

Fig. 2—Two states with energies E_j and E_i and spin populations n_j and n_i are shown. Spin-lattice relaxation times between states are indicated.

In Boltzmann equilibrium at temperature T, $dn_i/dt = 0$ and n_i and n_j assume their equilibrium values N_i and N_j. Thus

$$T_{ij}/T_{ji} = N_i/N_j = e^{(E_j - E_i)/kT}. \tag{2}$$

Although (2) has been derived for only a two-state system, it may be shown to apply to a multistate system by solving simultaneously all of the rate equations.

Spin-lattice interaction shortens the lifetimes of the states and thus broadens the resonance line. A review of spin-lattice interaction is given by Cooke[5] and Gorter.[6]

Spin Temperature

It is frequently convenient to retain the form of the Boltzmann relation (2) even when the spin system is

[3] M. H. L. Pryce and K. W. H. Stevens, "The theory of magnetic resonance—line widths in crystals," *Proc. Phys. Soc. A*, vol. 63, pp. 36–51; January, 1950.

[4] J. H. Van Vleck, "The dipolar broadening of magnetic resonance lines in crystals," *Phys. Rev.*, vol. 74, pp. 1168–1183; November, 1945.

[5] A. H. Cooke, "Paramagnetic relaxation effects," *Rep. Prog. Phys.*, vol. 13, pp. 276–294; 1950.

[6] C. J. Gorter, "Paramagnetic Relaxation," Elsevier Press, Amsterdam, The Netherlands; 1947.

not in thermal equilibrium with the lattice. If the populations of two spin states (Fig. 2) are n_i and n_j then a spin-temperature T_s is defined by

$$n_i/n_j = e^{(E_j-E_i)/kT_s}. \qquad (3)$$

That is, T_s is used as a measure of the ratio n_i/n_j even when thermal equilibrium is nonexistent. If

$$n_i > n_j \quad \text{then} \quad T_s > 0,$$
$$n_i = n_j \quad \text{then} \quad T_s = \pm\infty,$$
$$n_i < n_j \quad \text{then} \quad T_s < 0.$$

The last case that of negative spin temperatures may be caused by "population inversion." Negative spin temperatures are "hotter" than positive spin temperatures. From (3) one finds that

$$n_i - n_j = n_j\left[e^{(E_j-E_i)/kT_s} - 1\right]. \qquad (4)$$

Since $(E_j - E_i)/kT_s$ is usually small when dealing with microwave frequencies

$$\left[e^{(E_j-E_i)/kT_s} - 1\right] \approx (E_j - E_i)/kT_s \text{ and } n_j \approx \frac{N}{2}$$

and (4) becomes

$$n_i - n_j \approx \frac{N}{2}(E_j - E_i)/kT_s. \qquad (5)$$

Stimulated or Induced Emission and Absorption

A particle may exchange energy with an incident radiation field in accordance with the Bohr frequency condition

$$E_j - E_i = h\nu_j \qquad (6)$$

emitting or absorbing a photon of frequency ν_{ji} depending on whether $E_j >$ or $< E_i$; $h = 6.6 \times 10^{-27}$ erg seconds is Planck's constant.

This energy exchange is governed by certain transition probabilities W_{ij} which are products of transition probability coefficients w_{ij} and the square of the field amplitude. Thus

$$W_{ij} = w_{ij}H_{rf}^2 \qquad (7)$$

w_{ij} is a complicated function that depends on the nature of the states i and j as well as on the nature of the radiation field. The relevant field vector may be either E_{rf} or H_{rf}, H_{rf} is used in (7). Frequently $w_{ij} = 0$ in which case the transition is said to be "forbidden." As a consequence of the fact that the coefficient w_{ij} contains the square of a matrix element connecting states i and j,

$$w_{ij} = w_{ji}. \qquad (8)$$

That is, the probability for induced emission is equal to that for induced absorption. Consider the two-state system depicted in Fig. 3, radiation of frequency $\nu = (E_j - E_i)/h$ is incident.

The total power absorbed is $P_{ij} \propto n_i w_{ij} H_{rf}^2$, and the total power emitted, $P_{ij} \propto n_j w_{ij} H_{rf}^2$. Hence using (5), (6), and (8), the net power absorbed is

$$P_{abs} \propto \frac{N}{2}\frac{h\nu}{kT_s} w_{12} H_{rf}^2. \qquad (9)$$

So far it has been tacitly assumed that the energy levels are sharp, corresponding to zero width for the resonance line. In practice because of the broadening effects of the relaxation processes the resonance line will have some frequency spread $\Delta\nu$. As expected the broader the line the less the intensity of absorption. Thus (9) may be rewritten

$$P\text{ abs} \propto \left(\frac{N}{T_s}\frac{1}{\Delta\nu} w_{12}\right) H_{rf}^2 \qquad (10)$$

where the constants have been dropped. In this case the absorption is a magnetic one and macroscopically it would be represented as

$$P\text{ abs} \propto \chi'' H_{rf}^2 \qquad (11)$$

where χ'' is the imaginary component of the magnetic susceptibility of the material. Since the bracketed term in (10) depends only on the state of the material

$$\chi'' \propto \frac{N}{T_s}\frac{1}{\Delta\nu} w_{12}. \qquad (12)$$

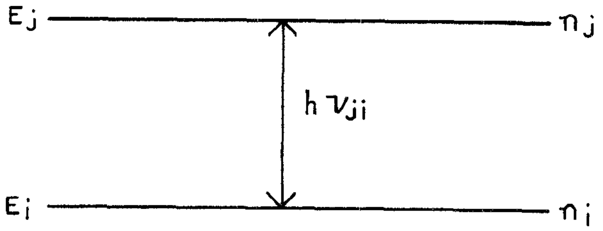

Fig. 3—Two states with energies E_j and E_i and spin populations n_j and n_i are shown. Radiation of frequency ν_{ji} is depicted as inducing transitions.

In thermal equilibrium, $T_s > 0$ and both Pabs and χ'' are positive corresponding to positive losses. In a maser, population inversion corresponding to $T_s < 0$ is produced and both Pabs and χ'' become negative corresponding to negative loss or gain. The stimulated emission may be shown in a more sophisticated treatment[7,8] to be coherent with respect to the incident radiation. Thus, in the case of a negative temperature material the amplified output signal is phase related to the input signal.

Spontaneous Emission

In addition to stimulated emission and absorption, there is the process of spontaneous emission which occurs in the absence of applied radiation. The mechanism may be considered to be that of stimulated emission induced by the zero point oscillations of the vacuum.

[7] L. I. Shiff, "Quantum Mechanics," McGraw-Hill Book Co., Inc., New York, N. Y.; 1955.

[8] W. Heitler, "The Quantum Theory of Radiation," Oxford University Press, New York, N. Y.; 1954.

Spontaneous emission tends to maintain thermal equilibrium between a particle system and the radiation field. This is in contradistinction to the nonradiative relaxation processes which tend to maintain equilibrium between particles.

The transition probability for spontaneous emission is $\propto \nu^3$. Although it is an important thermalizing process at optical frequencies, its effects are negligible compared with relaxation processes at microwave frequencies.

Spontaneous emission is, however, of great importance to a maser. Because it is radiative and independent of any incident radiation it is the ultimate source of noise in such a device.

Power Saturation

It is evident from (9) that resonance radiation tends to equalize populations. If $n_i > n_j$ there is a net absorption of energy and a net drift of particles from the lower to the higher state. Thus the population difference $n_i - n_j$ decreases. This process would continue until $n_i = n_j$ if it were not for the spin-lattice relaxation which tends to maintain Boltzmann equilibrium.

If the power level is low the relaxation process is predominant. At high-power levels, however, there will be appreciable departures from thermal equilibrium. In practice the populations may be essentially equalized. This is known as a condition of power saturation. A detailed discussion of this effect is given by Portis.[9]

The Three-Level Maser

The Three-Level Excitation Method

In a maser an emissive or negative temperature condition is produced. One way of achieving this continuously is the three-level method. This method was suggested by Basov and Prokhorov;[10] its application to the solid state was proposed by Bloembergen.[11]

Consider an ensemble of particles distributed over three energy levels as shown in Fig. 4.

Nonzero transition probability coefficients are assumed to exist between all states. Particles will tend to drift between states with the indicated spin-lattice relaxation times. In the absence of radiation-thermal equilibrium at the lattice temperature T will exist and the populations will be those of Boltzmann N_1, N_2, and N_3. Resonance radiation of frequency ν_{31} (henceforth called the "pump") will induce transitions between states 1 and 3 and disturb thermal equilibrium, and a new population distribution n_1, n_2, and n_3 will occur. If the pump is sufficiently powerful it will overcome relaxation processes; in practice populations n_1 and n_3 will be essentially equalized. The rate equation for n_2 becomes

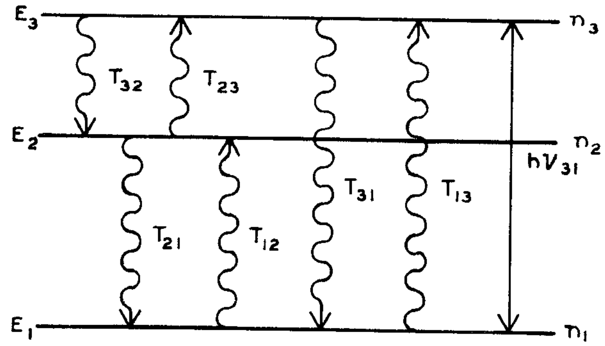

Fig. 4—Three states with associated energies, populations and spin-lattice relaxation times are depicted. Radiation induces transitions between states 1 and 3.

$$\frac{dn_2}{dt} = \frac{n_1}{T_{12}} + \frac{n_3}{T_{32}} - \frac{n_2}{T_{21}} - \frac{n_2}{T_{23}}. \tag{13}$$

In the steady state $dn_2/dt = 0$ and since $n_1 = n_3$

$$\frac{n_3}{n_2} = \frac{n_1}{n_2} = \frac{1/T_{21} + 1/T_{23}}{1/T_{12} + 1/T_{32}} \tag{14}$$

but $N_1/T_{12} = N_2/T_{21}$ and $N_2/T_{23} = N_3/T_{32}$ from (2), and (14) becomes

$$\frac{n_3}{n_2} = \frac{n_1}{n_2} = \frac{T_{32}/T_{21} + N_3/N_2}{\frac{T_{32}}{T_{21}}\frac{N_2}{N_1} + 1}. \tag{15}$$

It is seen that

$n_3/n_2 > 1$ or $n_2/n_1 > 1$ as the right-hand side $>$ or $<$ 1.

In either event gain will occur; in the first case for radiation of frequency ν_{32}, in the second case for radiation of frequency ν_{21}. It is possible theoretically for $n_3/n_2 = n_2/n_1 = 1$ but this requires a most unlikely combination of parameters in (15).

For high gain a large population inversion is required. Two extreme cases may be distinguished.

1) $T_{32}/T_{21} \approx 1$. Appreciable inversion will occur if N_3/N_2 differs greatly from N_2/N_1. Since these Boltzmann ratios are related to the frequencies ν_{32} and ν_{21} by equations of the form (2) and (6), it is seen that there must be a large ratio for the frequencies ν_{32} and ν_{21}. Further, the negative spin temperature will occur at the lowest frequency. This in turn requires a large ratio of pumping to signal frequencies. Although this approach is perhaps useful at low frequencies because of its simplicity, it has definite disadvantages in a high-signal frequency application.

2) $N_3/N_2 \approx N_2/N_1$ that is $\nu_{32} \approx \nu_{21}$. Appreciable inversion will occur only if T_{32} differs greatly from T_{21} and the negative temperature and gain will occur at that frequency with the longest associated relaxation time. Somewhat meager experimental data indicates that such relaxation times are usually about equal. One method

[9] A. M. Portis, "Electronic structure of F centers; saturation of the electron spin resonance," *Phys. Rev.*, vol. 91, pp. 1071–1078; September, 1953.

[10] N. G. Basov and A. M. Prokhorov, "Possible methods of obtaining active molecules for a molecular oscillator," *J. Exper. Theoret. Phys. USSR*, vol. 1, p. 184; July, 1955.

[11] N. Bloembergen, "Proposal for a new type solid state maser," *Phys. Rev.*, vol. 104, pp. 324–327; October, 1956.

for producing large relaxation time ratios has been demonstrated.[12] A small amount of paramagnetic impurity is introduced into the lattice in such a way that its resonance frequency is equal to one of the resonant frequencies of the active material. Resonant spin-spin interaction and energy exchange then occurs between the impurity and one pair of levels of the active material but not for the other pair. This mechanism effectively "short circuits" one relaxation time without affecting the other. This doping technique would appear to be of value in a high-frequency maser. Although it puts an additional restriction on the choice of material, this does not appear prohibitive.

In both of the above cases it is evident that if appreciable population inversion is to occur then both N_3/N_2 and N_2/N_1 should be small. This suggests the use of low-bath temperatures. It is evident that the two frequencies ν_{32} and ν_{21} should not be exactly equal. Otherwise a signal would simultaneously see gain and loss, a negative temperature at one transition being accompanied by a positive temperature at the other.

Some remarks are in order regarding the applicability of the simple theory of the three-level excitation method as outlined above. The populations n_1 and n_3 may never be exactly equal. The presence of a strong signal at either ν_{32} or ν_{21} will affect the population distribution. Both effects may be taken into account by solving simultaneously the three-rate equations for n_1, n_2, and n_3 including the radiation. For instance a strong signal at the amplifying frequency will reduce the population inversion and hence the gain—the device is self-limiting.

At first sight the frequency relationship $\nu_{21}+\nu_{32}=\nu_{31}$ may suggest that the device operates because of nonlinearities. If, however, the device operates as described here, this cannot be the case. For instance, it has not been necessary to take into account the phase of the radiation. Furthermore, if the device is in an amplifying condition then the pump may be turned off and amplification will persist for a time determined by the spin-lattice relaxation. However, it is possible to drive such a three-level system so that it behaves in a nonlinear fashion. This condition has been investigated by Javan,[13] Clogston,[14] and Prokhorov.[15] In order to achieve nonlinear operation, it is necessary for the radiation to be sufficiently powerful to overcome the disordering effects of spin-spin interaction and thereby create coherence in the spin system. Under these conditions, mixing and conversion gain are possible. Such a device would appear to be closely related to a parametric-type amplifier. In order for the linear theory given here to be applicable the pump must be sufficiently powerful to overcome the spin-lattice relaxation only and produce power saturation, but it must not be so powerful as to overcome spin-spin relaxation and produce a coherent ordering of the spin system. Essentially, this imposes a restriction on the material namely the spin-spin relaxation time must be much shorter than the spin-lattice relaxation time.

Some Remarks on Paramagnetic Salts

Ionically bound paramagnetic salts appear to be a particularly fruitful source of suitable materials for a three-level solid-state maser. The theory of such materials has been reviewed by Bleaney and Stevens[16] and experimental data on them has been reviewed by Bowers and Owen.[17]

When an atom enters into chemical combination the magnetic moments associated with its valence electrons are usually "paired off" resulting in a diamagnetic structure. Certain elements belonging to the "transition groups" have incomplete inner electron shells. When these atoms enter into a chemical compound the magnetic moments associated with the inner shells are not always "paired off" and a paramagnetic material results.

The relevant inner shells are those of the 3d, 4d, 5d, 4f, and 5f electron configurations. The corresponding groups are known as the iron, palladium, platinum, rare earth, and transuranic groups. In maser applications only the iron and rare earth groups would appear to be attractive. The palladium and platinum groups have a strong tendency to form diamagnetic covalent complexes and it is difficult to control the spin concentration by magnetic dilution methods in the few remaining paramagnetic materials. The transuranic group is for the most part strongly radioactive.

The number of useful ions is further reduced by the requirement of having at least three low-lying and therefore appreciably populated energy levels. Effectively this leaves Ni^{++}, Cr^{+++}, Fe^{+++}, Gd^{+++}, Mn^{++}, and V^{++}. The last two ions exhibit extensive hyperfine structures and except in extremely wide-band applications this has the effect of reducing the useful spin concentration.

The paramagnetic ion in the solid state is subject to strong crystalline electric field gradients. These fields produce strong perturbations of the electronic orbital motion. The electron spin is coupled to these orbits by spin-orbit coupling. The resulting states are complicated admixtures of orbital and spin-wave functions.

Fortunately, the magnetic behavior of the lowest lying group of states may be described by the use of a "Spin-Hamiltonian" which requires no detailed knowledge of these effects.

Essentially one may ignore the correct but complicated wave functions and simply characterize the lowest

[12] G. Feher and H. E. D. Scovil, "Electron spin relaxation times in gadolinium ethyl sulphate," *Phys. Rev.*, vol. 105, pp. 760–762; January, 1957.

[13] A. Javan, "Theory of a three level maser," *Phys. Rev*, pp. 1579–1589; September, 1957

[14] A. M. Clogston, "Susceptibility of the three level maser," *J. Phys. Chem Solids*, in press.

[15] A. M. Prokhorov, "Theory of the Three Level Maser," paper presented at URSI Meeting, Boulder, Colo; September, 1957

[16] B. Bleaney and K. W. H Stevens, "Paramagnetic resonance," *Rep. Prog. Phys.*, vol. 16, pp. 108–159; 1953.

[17] K. D Bowers and J. Owen, "Paramagnetic resonance II," *Rep. Prog. Phys.*, vol. 18, pp. 304–373; 1955.

lying group of states by a single quantum number S' known as the "effective spin." S' is defined by equating $2S'+1$ to the number of energy levels making up the ground state. In special cases S' is equal to the true-spin S but this is not necessarily so.

The ground state is now treated as a magnetic dipole which may assume $2S'+1$ orientations in an applied magnetic field. The effective magnetic moment of this dipole depends upon the correct wave functions but may easily be found by experiment. The effective spin is however not free since, unlike a real spin, it is affected by the crystalline electric field which produces a "Stark splitting" of the energy levels. This Stark energy must be taken into account along with the magnetic energy when describing the behavior of the states. The Hamiltonian $\mathcal{H}$ thus consists of a sum of magnetic and Stark energy operators which are to be applied to the effective spin states. The entire magnetic behavior of the material may then be described simply by specifying S' and a few constants.

The magnetic energy or Zeeman operator is simply $g \cdot \beta H \cdot S$ where β is the Bohr magneton $= 9.21 \times 10^{-21}$ Gauss cm^3 and g is the "spectroscopic splitting factor" or "effective g." $g\beta S$ may be termed the effective magnetic moment. In most maser applications g will have a value near 2 and be almost isotropic.

Assuming that the crystalline potential is an electrostatic one it may be expanded in spherical harmonics as

$$V = \sum_{nm} A_n{}^m r^n Y_n{}^m(\theta, \phi).$$

To find the corresponding Stark energy operators one may follow a method given by Stevens.[18] A brief heuristic treatment is given here.

First of all one observes that the number of operators is finite. This is because the electron wavefunctions may also be expanded in spherical harmonics and d and f wavefunctions do not contain harmonics for which $n > 4$ and 6, respectively. When forming the integral of the matrix elements of V all terms, for which $n > 4$ and 6, respectively, vanish by the orthogonality relations for spherical harmonics. Similar arguments show that only terms for which n is even need be retained. The term $n = 0$ may be dropped since it is just an additive constant. Further reduction occurs by taking into account the symmetry properties of the crystal. Finally, when looking for operators to be applied to states of the effective spin S' it is necessary to retain only those operators that have matrix elements spanning the manifold of S'. The remaining spin operators are now formed in such a manner that they transform under rotation as do the corresponding terms of the crystalline field expansion.

For most maser applications the following Hamiltonian provides an adequate description, where $S\pm = S_x \pm iS_y$. S, S_x, S_y, and S_z are the usual spin operators and may be considered to be the quantum equivalent of the corresponding classical spin mechanical momenta.

$$\mathcal{H} = g \cdot \beta H \cdot S + D[S_Z{}^2 - 1/3S(S + 1)] + 1/2E[S_+{}^2 + S_-{}^2] \quad (16)$$

where

S is the effective spin (the prime being dropped),
g and β have already been described,
D and E are constants determining the strength of the Stark interaction,
g, D, and E are conveniently found by experiment. Values for various materials are given by Bowers and Owen.[17]

The operator $[S_Z{}^2 - \frac{1}{3}S(S+1)]$ represents a term of axial symmetry and corresponds to the harmonic $Y_2{}^0$. The operator $[S_+{}^2 + S_-{}^2]$ represents a term of rhombic symmetry and corresponds to the harmonic $Y_2{}^2$. Although additional terms may exist they are usually small.

The ions Ni^{++}, Cr^{+++}, Fe^{+++}, and Gd^{+++} have effective spins 1, 3/2, 5/2, and 7/2 respectively.

The material may be utilized in several ways. Some of these are now illustrated.

Case 1—H Applied Parallel to the Symmetry Axis Z, $S = 3/2$: Consider initially only the first two terms in $\mathcal{H}$. The energy levels as a function of H are as shown in Fig. 5, opposite. For $H = 0$ there is an initial splitting into two pairs of degenerate states by the Stark field—a positive sign is assumed for D. The usual type of magnetic dipole transitions corresponding to the selection rule $\Delta S_Z = \pm 1$ are permitted. These transitions require that the rf magnetic field be perpendicular to H as in the classical gyroscopic model. It is seen that three energy levels with the requisite transitions exist in the region near point A. At higher magnetic fields, above "crossover" maser operation may not occur since although the energy levels are available no "double jump" transitions are permitted for the pump.

Consider now the addition of the last term in $\mathcal{H}$. The term $\frac{1}{2}E[S_+{}^2 + S_-{}^2]$ has only off diagonal matrix elements. It will have two effects. Depending on its magnitude it will shift the positions of the energy levels. However, it will also admix spin states differing by $\Delta S_Z = \pm 2$; *e.g.*, the original state $S_Z = -3/2$ will now be an admixture of the states $S_Z = -3/2$ and $S_Z = +1/2$. As a result of this admixture transitions of the $\Delta S_Z = 0$ type are permitted when the rf magnetic field is parallel to H. These transitions correspond to double jumps and hence the requisite transitions for both signal and pump are now available in the region above crossover. In the high-field region the double jump transition probability is $\propto (E/g\beta H)^2$.

Case 2—H Applied Perpendicular to the Symmetry Axis, $S = 3/2$: Consider again only the first two terms in $\mathcal{H}$. It is convenient to rewrite the Hamiltonian retaining H as the Z direction. The transformed Hamiltonian becomes

$$\mathcal{H} = g\beta HS_Z - 1/2D[S_Z{}^2 - 1/3S(S + 1)] + 1/4D[S_+{}^2 + S_-{}^2]. \quad (17)$$

[18] K. W. H. Stevens, "Matrix elements and operator equivalents connected with the magnetic properties of the rare earth ions," *Proc. Phys. Soc. A*, vol. 65, pp. 209–215; March, 1952.

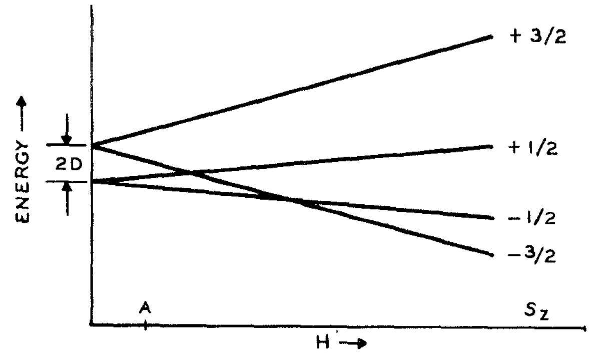

Fig. 5—Energy levels are shown as a function of magnetic field H when H is parallel to the crystalline symmetry axis. $S=\frac{3}{2}$.

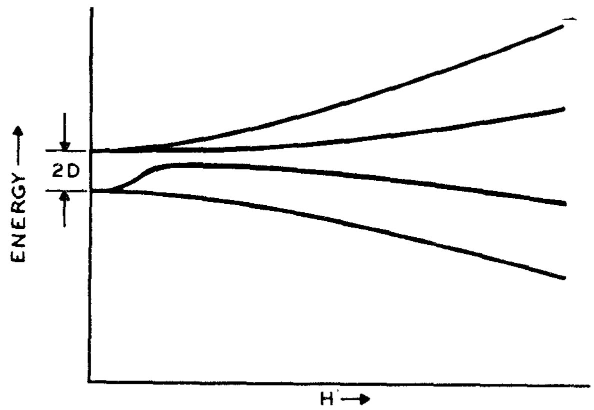

Fig. 6—Energy levels are shown as a function of magnetic field H when H is perpendicular to the crystalline symmetry axis. $S=\frac{3}{2}$.

The energy levels as a function of H are shown in Fig. 6. The requisite energy levels are available and so are the required transitions. It will be noted that the application of the magnetic field perpendicular to the symmetry axis has in effect created a term $[S_+^2+S_-^2]$ even though there is no term E of rhombic symmetry in the crystalline field. The resulting admixture again permits transitions of the $\Delta S_Z=0$ type. This transition probability is $\propto (D/g\beta H)^2$ in the high-field region.

Case 3—$H=0$, $S=1$: If all the terms in $\mathcal{H}$ are zero the ground state has a threefold degeneracy in spin. The term $[S_Z^2-\frac{1}{3}S(S+1)]$ partially removes the degeneracy leaving the states $S_Z=\pm 1$ still degenerate as shown in Fig. 7. The term $[S_+^2+S_-^2]$ removes the remaining degeneracy. All transitions are permitted. This particular mode of operation has been suggested by Bowers and Mims[19] and has the advantage that no applied magnetic fields are required to produce the requisite energy level scheme.

It is apparent that materials must be chosen for a particular mode and frequency of operation. In particular crystalline field parameters must be chosen such that the appropriate transition probabilities are sufficient. Materials with a wide range of D and E are available to make this possible.

Once the material, frequency and mode of operation

[19] K. D. Bowers and W. B. Mims, "Three Level Maser Without a Magnetic Field," paper presented at Conference on Electronic Tube Research, Berkeley, Calif.; June, 1957.

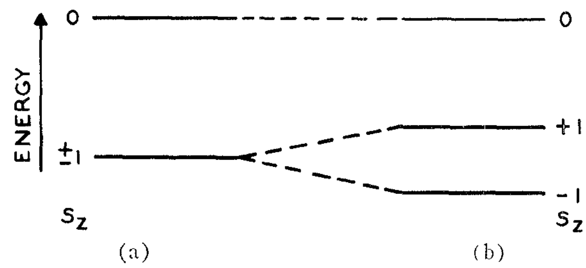

Fig. 7—Energy levels for $S=1$ are shown. (a) illustrates the effect of the axial terms $[S_z^2=\frac{1}{3}S(S+1)]$ only. (b) illustrates the combined effect of the axial term and a rhombic term $[S_+^2+S_-^2]$.

are fixed, the transition probability coefficients w_{ij} may be computed by standard techniques cf Bleaney and Stevens[16] and the susceptibility evaluated.

In addition to having the requisite energy levels and transition probabilities the material must satisfy additional requirements. The spin-lattice relaxation time must be fairly long in order for the pump to produce power saturation with reasonable powers. The ions Ni^{++}, Cr^{+++}, Fe^{+++}, and Gd^{+++} normally have sufficiently long relaxation times at liquid helium temperatures.

In some materials crystalline field dislocations may produce appreciable inhomogeneous broadening of the resonance line. Inhomogeneous broadening from either crystal dislocations or nonresonant spin-spin interaction appears undersirable. Computations by Mims[20] show that under certain conditions of inhomogeneity a three-level maser will fail to exhibit gain. It appears desirable to increase the concentration of active spins to the point where homogeneous spin-spin broadening is the dominant line width determining mechanism. Further increase in concentration should be limited by bandwidth requirements since the pumping power is increased along with the concentration. In order to control the spin concentration, it is necessary to select a material such that it may be "magnetically diluted" with an isomorphous diamagnetic element; *e.g.*, diamagnetic Al^{+++} is frequently used to dilute paramagnetic Cr^{+++}.

Some Design Considerations

Design Considerations Relating to the Material

1) Gain—the intrinsic gain is proportional to χ''. Eq. (13) indicates that a high-spin concentration is desirable and that $\Delta\nu$ should be small. These two requirements are not independent. There is always some minimum $\Delta\nu$ arising from inhomogeneous broadening. Once the spin concentration is increased to the point where homogeneous broadening between active spins is the line width determining mechanism $\Delta\nu$ is approximately proportional to N and further increase of N has little effect on the gain. Relation (13) indicates that a small negative Ts is required—this infers a low bath temperature and appropriate relaxation time and frequency ratios.

[20] W. B. Mims, private communication.

2) Bandwidth—the intrinsic bandwidth is approximately proportional to $\Delta\nu$ if homogeneous broadening is predominant.
3) Pumping power—this increases as the spin-lattice relaxation time decreases and suggests the desirability of low bath temperatures. It increases with the spin concentration and consequently excessive concentrations should not be used. The pump transition probability shou'd be sufficiently high for the pump to be effectively matched to the material losses. If liquid helium is used as a refrigerant, pumping powers of a few milliwatts appear reasonable since liquid helium boils off at the rate of about 1 cc per hour for a power absorption of 1 mw.
4) Output Power—this is limited by saturation effects but will normally be quite sufficient if the device is used as a low-noise preamplifier.
5) Noise—several authors[21–23] have treated the subject of noise in masers. The noise of the device may be expressed in terms of kT_nB where T_n is the effective noise temperature. The computations indicate that the ultimate noise is that arising from spontaneous emission and that the ultimate noise temperature is just equal to the magnitude of the effective spin-temperature $|T_s|$. This again suggests the desirability of low-temperature operation since in a three-level maser large population inversions result only if the bath temperature is low. Additional noise may arise from resistive losses in the microwave circuit particularly from any parts which are at room temperature. With proper design it should be possible to keep this last source of noise small.
6) Refrigerant—the best results are obtained at helium temperatures since gain, bandwidth, noise, and pumping power requirements all deteriorate as the bath temperature increases. In some system applications it may be desirable to operate at higher temperatures. Liquid hydrogen because of its ease of handling and high-latent heat appears very attractive.
7) Linearity—as previously mentioned, linear operation imposes the restriction that spin-spin relaxation time be much shorter than spin-lattice relaxation time. Materials which satisfy normal bandwidth and reasonable pumping power requirements will automatically satisfy this condition.

Design Considerations Relating to the Microwave Circuit

First of all it is important, particularly from the noise aspect, to keep all losses in the microwave circuit to a minimum. The simple maser, like a negative resistance device, will amplify for both directions of propagation. It is important, therefore, to provide some type of nonreciprocal isolation between the interaction medium and the output load. Since the latter in most cases will be at room temperature its available noise power $kT_{290}B$ would otherwise enter and be amplified by the medium and return as amplified noise to the load, the result would be a minimum noise temperature of 290° for the entire device.

Conceptually, perhaps the simplest maser circuit is just a waveguide filled with active material. In this case, an input signal increases exponentially as it travels down the guide. For reasonable negative imaginary susceptibilities, however, such a system would have to be several tens of meters long in order to provide useful gain. Evidently a slow-wave structure is required so that the wave may interact for a longer time with a given amount of material.

The extreme case of a slow-wave structure is a simple cavity. Alternatively the cavity may be looked upon as a means of supplying regeneration or positive feedback, the amount of feedback being dependent on the cavity Q. The cavity provides a simple method of obtaining high gain in a relatively small volume. The bandwidth of such a device is initially limited by the cavity Q and then further reduced by the application of the positive feedback. A further disadvantage exists because the large amount of regeneration normally employed makes the device quite unstable at high gains. Frequently, a reflection type cavity is used, the input and output signals being separated by a circulator which also serves to decouple the noisy room temperature load from the interaction medium. In practice a cavity resonant at both the signal and pumping frequencies is employed, the resonant cavity for the pump serving to match the pumping power into the positive imaginary susceptibility associated with the pumping frequency.

A compromise between the waveguide and the cavity is an iterated structure. Such a maser may be made unidirectional and hence stable with the use of gyromagnetic material. The nonreciprocal material may be a ferrite or perhaps the active material itself since the classical gyroscopic model is derived from quantum mechanical transitions of the $\Delta S_Z = \pm 1$ type which have been shown to occur in paramagnetic salts. Perhaps the most elementary broad-band structure is simply an iterated linear array of low Q cavities and ferrite isolators. The most elegant type of structure appears to be a continuously loaded single periodic structure. The design of such a structure must be compatible with the requirement that the applied magnetic field must have specific orientations with respect to the signal and pumping rf fields. A further restriction is that it must exhibit nonreciprocal properties and the magnetic field applied to the active material should be the same field that is responsible for the nonreciprocity.

The interaction material and the corresponding

[21] K. Shimoda, H Takahasi, and C. H. Townes, "Fluctuations in amplification of quanta with application to maser amplifiers," *J. Phys. Soc. Japan*, vol. 12, pp. 686–700; June, 1957.

[22] R. V. Pound, "Spontaneous emission and the noise figure of maser amplifiers," *Ann. Phys.*, vol. 1, pp. 24–32; April, 1957

[23] M. W. P. Strandberg, "Inherent noise of quantum—mechanical amplifiers," *Phys. Rev.*, vol. 106, pp. 617–620; May, 1957.

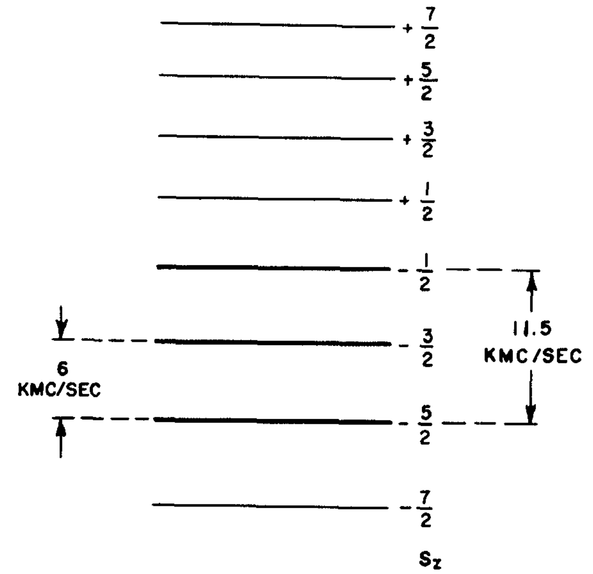

Fig. 8—The energy levels of Gd^{+++} in the ethyl sulfate for a large magnetic field applied perpendicular to the symmetry axis. The heavy lines identify the maser levels.

structure must be immersed in the refrigerant. Standard low temperature techniques are available for this.

Some Design and Performance Characteristics of a Particular Device

Design

A brief description of a 9-kmc three-level cavity maser developed at Bell Telephone Laboratories, has appeared in the literature.[24] The 6-kmc device described here is essentially a scaled version.

The material used is a lanthanum ethylsulphate crystal containing $\frac{1}{2}$ per cent gadolinium and $\frac{1}{5}$ per cent cerium. La^{+++} being diamagnetic acts as a magnetic diluting agent to reduce spin-spin interaction. Gd^{+++} acts as the active paramagnetic. Ce^{+++} acts as a spin-lattice relaxation time doping agent.

Gadolinium ethylsulphate was chosen because there was sufficient information available[25,26] to show that it possessed the requisite properties. In a practical application its slight chemical instability and its high room temperature dielectric loss are disadvantages.

Gd^{+++} being in a pure-spin state has an effective spin S' equal to the true-spin $S=7/2$. The magnetic behavior is described to a close approximation by the Hamiltonian (16) with $g\approx 2$, $D=0.02$ cm^{-1} and $E=0$. Although there are a total of eight low-lying energy levels only three adjacent energy levels are used as shown in Fig. 8. A field of $\sim$1800 ϕ is applied perpendicular to the symmetry axis and the mode of operation

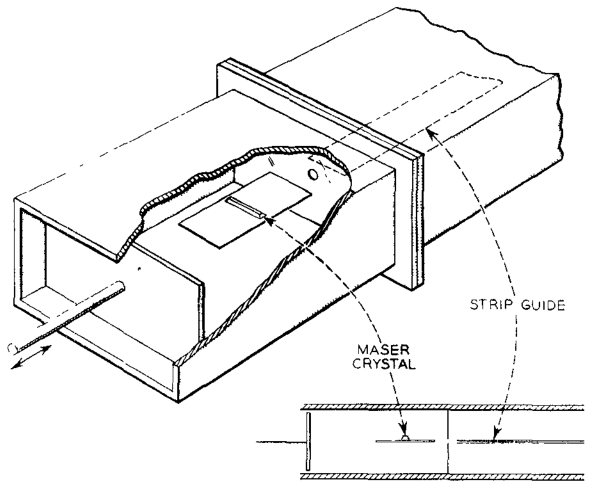

Fig. 9—The pump and signal cavity configuration is illustrated. The rectangular waveguide carrying the pump terminates in a $(\frac{3}{2})$ λ rectangular cavity. The signal propagates along the strip line. A $(\frac{1}{2})$ λ length of strip line placed inside the rectangular cavity acts as the signal cavity.

used is that described under Case 2) above.

The gadolinium ion experiences an inhomogeneous spin-spin broadening of $\sim$20 mc from the magnetic moments of the protons in the surrounding water molecules. Consequently the gadolinium spin concentration was increased to $\frac{1}{2}$ per cent so that the resulting homogeneous spin-spin relaxation time $\approx 10^{-8}$ seconds corresponding to a line width of ≈ 30 mc was predominant. The spin-lattice relaxation time of the unperturbed transitions is $\sim 10^{-4}$ seconds at the temperature of liquid helium and hence the material satisfies the requirements for linear operation. At 1800 ϕ the energy levels are fairly equally spaced and reliance must be placed on large spin-lattice relaxation time ratios to obtain appreciable population inversion. It has been found experimentally[12] that $\frac{1}{5}$ per cent cerium provides adequate preferential homogeneous spin-spin interaction for a sufficient ratio.

The interaction cavity was not sealed against liquid helium for experimental convenience. Liquid helium has an appreciable dielectric constant and when boiling at 4°K at atmospheric pressure the resulting bubbles modulate the cavity frequency. In order to prevent this the vapor pressure and hence the temperature was reduced by pumping, since bubble formation does not occur below the λ point. A temperature of 1.2°K was used.

A reflection-type cavity is used for the microwave circuit; the input and output signals being separated by a circulator at room temperature. The particular cavity geometry used is shown in Fig. 9. Pumping power at ≈ 11.5 kmc enters through an X-band waveguide which is terminated in a tunable rectangular cavity resonant in the $(3/2)\lambda$ mode. The signal propagates along the strip line. Inside the rectangular pumping cavity is a $(1/2)\lambda$ (at 6 kmc) length of a strip line which forms the signal cavity. The crystal is mounted at the rf magnetic field maximum in the center of the strip cavity. This in turn is so positioned that the crystal also sees an rf mag-

[24] H. E. D. Scovil, G. Feher, and H. Seidel, "Operation of a solid state maser," *Phys. Rev.*, vol. 105, pp. 762–763; January, 1957.
[25] B. Bleaney, H. E. D. Scovil, and R. S. Trenam, "The paramagnetic resonance spectra of gadolinium and neodymium ethyl sulphates," *Proc. Roy. Soc. A*, vol. 223, pp. 15–29; April, 1954.
[26] H. A. Buckmaster, "$\Delta M=\pm 2$ transitions in dilute gadolinium ethyl sulphate," vol. 34, pp. 150–151; January, 1956.

netic field maximum of the pump.

The signal, associated with a $\Delta S_Z = \pm 1$ transition, requires that the signal rf magnetic vector be perpendicular to the large applied dc field. The pump, associated with a $\Delta S_Z = 0$ transition, requires that its magnetic vector be parallel to the dc field. In the cavity geometry employed the signal, and pump rf magnetic fields are parallel to each other. The applied dc magnetic field is oriented at an angle of 45° to the plane of the strip in order to have rf field components satisfying the two transition probability requirements.

Performance

This device was constructed for some initial feasibility tests and its performance should not be taken to indicate the limits of performance which may be expected in a more practical device.

A pumping power of 38 mw was employed, probably less than 1 per cent of this being absorbed by the crystal. The discrepancy arises from the fact that a low pumping cavity Q provides a poor match. The pumping requirements may be materially decreased by better design.

With a gain of 20 db the bandwidth is ≈ 100 kc. Although the material has a line width of 30 mc this inherent bandwidth is not utilized because of the high $Q(\approx 6000)$ of the signal cavity and the associated positive feedback. The regeneration also results in a fairly small stability margin. Both bandwidth and stability would be improved if an appropriate unidirectional slow wave structure was employed.

The effective noise temperature of the entire device including circulator and monitoring equipment was $\sim$150°K. Most of this noise arose from the excessive, and easily reducible, room temperature losses of the circulator and monitoring equipment. The noise temperature of the actual maser was <35°K compared with the theoretical value of 4°K (obtained from a spin temperature of -4°K). The accuracy of the noise measuring apparatus was insufficient to allow identification of the true maser noise.

Nonmechanical Beam Steering by Scattering from Ferrites*

M. S. WHEELER†

***Summary*—A small aperture radiating circularly polarized energy is loaded with a spherical ferrite to produce an electronic beam directing system. The ferrite is immersed in a static magnetic field which is in general at an oblique angle with the undeflected direction of radiation. It is shown that radiation is principally in the direction of the magnetic field when the polarization is in the negative sense. From symmetry this allows beam deflection with two degrees of freedom.**

To consider an application for such a device, it is proposed that this deflection system be used in conical scan. A mechanization is shown which solves the problem in principle, but it is not competitive with present mechanical scanners from the point of view of side lobes, etc.

Introduction

USE is continually being made of two ferrite properties, the Faraday rotation and resonant absorption, to produce the many microwave components used today in waveguides and antennas. When these principles are applied to beam steering in antennas, it would seem that the properties of the ferrite are used in an indirect manner. Alternately, it should be possible to use ferrites directly to produce a deflection as the energy is passed through and over the ferrite. Such an effect was reported by Angelakos and Korman.[1]

It would be required that the ferrite be immersed in a large fraction of the radiated energy, and yet the ferrite must be relatively free from boundaries in such a manner that the energy would be free to change direction. The asymmetry, which is controlled and used to change beam direction, would be the relatively static saturating magnetic field to which direction all of the anisotropic properties of the ferrite are referred.

The advantages of such a beam-steering device are obvious: no mechanically moving parts are necessary. Less control power would be required, low-temperature starting would cease to be troublesome, and higher operating speeds could be achieved with a ferrite as compared to a corresponding mechanical deflection system. However, to be practically useful, it would be required that a ferrite scatterer have low dissipative loss, that it be made relatively reflectionless, and that a sufficient degree of beam perfection be achieved considering cross polarization and extraneous side lobes.

* Manuscript received by the PGMTT, May 20, 1957; revised manuscript received, August 5, 1957.

† Westinghouse Electric Corp., Air Arm Div., Baltimore 3, Md.

[1] D. J. Angelakos and M. M. Korman, "Radiation from ferrite filled apertures," Proc. IRE, vol. 44, pp. 1463–1468; October, 1956.